

OCTOBER 29, 2012

SECTION 7.7 - COMPLEX NUMBERS ~~AREA~~
(IMAGINARY NUMBERS)

Key
NOTES

* $i = \sqrt{-1}$

* $i^2 = -1$

EX = $\sqrt{-4} = \sqrt{-1} \cdot \sqrt{4}$
 $= i \cdot 2$
 $= 2i$

EX = $\sqrt{9} = 3i$ (short cut)

EX = $\sqrt{-27} = 3i\sqrt{3}$ or $(3\sqrt{3})i$

$a + bi$
↑ ↑
real imaginary
part part

EX = $3 + 2i$

Addition & Subtraction

EX. $(2 + 3i) - 5(2 - 4i)$
 $2 + 3i - 10 + 20i$
 $-8 + 23i$ ANS

Multiplication

EX. $\sqrt{-9} \sqrt{-4} \neq \sqrt{36}$
 $\sqrt{-9} \sqrt{-4}$
 $3i \cdot 2i$

* $6i^2 = 6 \cdot -1 = -6$

See above

EX. $\sqrt{-3} \sqrt{3} \sqrt{-4}$

$\sqrt{3} \cdot i \cdot \sqrt{3} \cdot i \cdot 2i$
 $3i^2 \cdot 2i$
 $3(1) \cdot 2i$
 $-6i$ ANS.

ex. $(3-2i)(4+5i)$

$$12 + 15i - 8i - 10i^2$$

$$12 + 7i - 10i^2$$

$$12 + 7i \overset{-1}{+10} \\ \boxed{22 + 7i} \text{ ANS}$$

ex. $(7-3i)^2$
 $(7-3i)(7-3i)$

$$49 - 21i - 21i + 9i^2$$

$$49 - 42i + 9(-1)$$

$$49 - 42i - 9 \\ \boxed{40 - 42i} \text{ ANS}$$

DIVISION
 *Rationalizing

ex. $\frac{3}{i} \cdot \frac{i}{i} = \frac{3i}{i^2} = \frac{3i}{-1} = \boxed{-3i} \text{ ANS}$

ex. $\frac{3}{2i} \cdot \frac{i}{i} = \frac{3i}{2i^2} = \frac{3i}{2(-1)} = \frac{3i}{-2} = \boxed{-\frac{3i}{2}}$

ex. $\frac{2}{3-i} \cdot \frac{3+i}{3+i} = \frac{2 \cdot 3+i}{9+3i-3i-i^2} = \frac{6+2i}{9-(-1)} = \frac{6+2i}{10}$
 conjugate

$$\frac{6}{10} + \frac{2}{10}i$$

$$\boxed{\frac{3}{5} + \frac{1}{5}i} \text{ ANS}$$

POWERS of i

$$i^1 = i$$

$$-i^2 = -1$$

$$-i^3 = i^2 \cdot i = (-1)(i) = -i$$

$$1i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$$

$$i^5 = i^4 \cdot i = 1 \cdot i = i$$

$$-i^6 = i^4 \cdot i^2 = (1)(-1) = -1$$

ex. $i^{20} = (i^2)^{10}$
 $= (-1)^{10}$
 $= 1$

ex. $i^{23} = i^{22} \cdot i$
 $= (i^2)^{11} \cdot i$
 $= (-1)^{11} \cdot i$
 $= -1 \cdot i$
 $= -i$

ODD POWER = - (neg) EVEN POWER = + (pos)